ON THE USE OF THE FALLING PLATE OSCILLOGRAPH AS A PHASE METER.

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The wave form of a periodic quantity is the curve which shows the magnitude of the quantity for each instant of time. It is always interesting and careful examination reveals relations that could hardly be discovered in any other way. In alternating current calculations, however, little can be done until the wave form is known accurately. There are two general methods in use, by which it may be determined—the *point to point method* and the *oscillograph* method. In the first, the quantity is measured by a meter, through which the circuit is closed, by a revolving contact-maker, for an instant at any part of the wave for which it may be set. The meter then indicates the value of the quantity at that particular point only. By taking such readings at various points in the cycle, the whole wave may be plotted. As this process is somewhat laborious, various instruments, called *wave tracers*, have been designed to facilitate the operation. In the *Rosa curve tracer* a double potentiometer is used. The operator fixes his eye on the galvanometer, and produces balance by means of a small crank, which turns the cylinder carrying the potentiometer wire. When this occurs, a second lever is pulled, which automatically prints a point of the curve on a paper fixed in the proper position, and also turns the contact-maker to the next position.

Either of the foregoing methods requires considerable time to plot a whole curve. The successive points are obtained from different waves. For example, a good operator can get a curve in five minutes if the instrument is in order. If he is working on a sixty-cycle circuit, he has obtained his curve from 18,000 successive waves. It will be a true curve, therefore, if he has kept his conditions absolutely constant in the interval. This is always troublesome to do, but particularly so in commercial work where the operator seldom has control of the generator. To avoid this difficulty, the oscillograph has been devised, by which it is possible to obtain the form of a single wave, or a number of successive waves.

The oscillograph is essentially a galvanometer of very short period. The one used in this work as shown in Fig. 1 is of the moving coil type, made under the Duddell patents. The field is

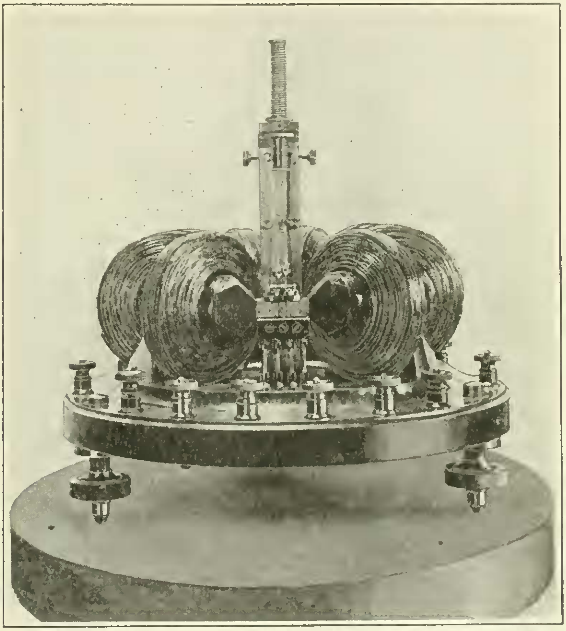

FIG. 1.

supplied by an electromagnet, the coils of which are wound in several sections, so that different voltages may be used for the exciting current. The normal current nearly saturates the core, so that slight changes in the value of the current do not cause appreciable changes in the strength of the field. The coil consists of an inverted U with the ends rigidly fastened at the bottom by a rubber block, and connections made to the binding posts. The upper loop is threaded over a small pulley, to which is fastened the spring by means of which the tension is applied to the strips. There are two distinct loops, thus permitting the taking of two curves simultaneously. The free period is approximately one ten-thousandth of a second, undamped. There are three mirrors, one fastened on each coil, and one fixed in the center, to give a zero line. The maximum current used is about one tenth ampere.

To obtain a curve, it is necessary to provide uniform motion perpendicular to the motion of the mirrors. In this instrument it is accomplished by means of a falling photographic plate. This

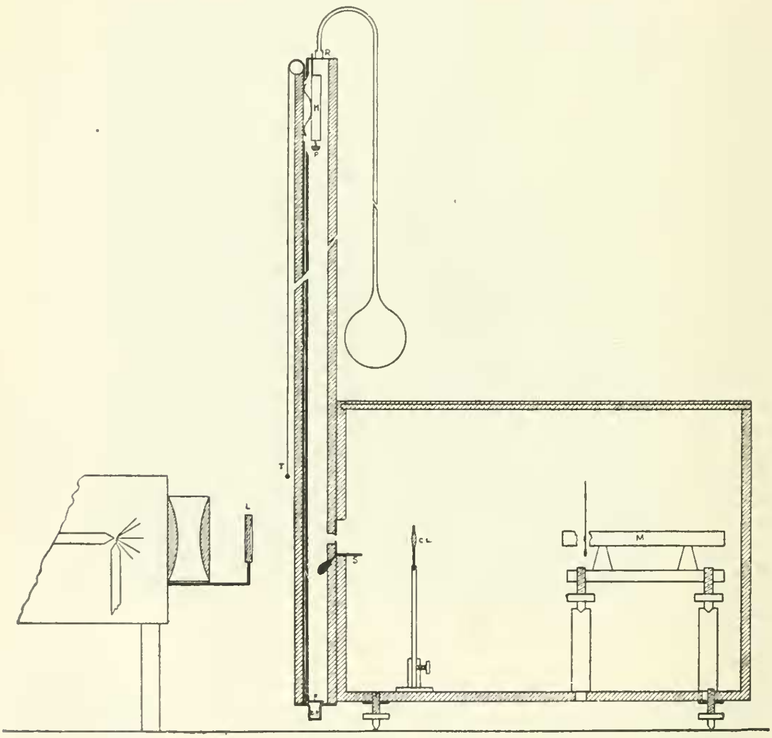

FIG. 2.

motion is uniformly accelerated, but the error in the length of the plate is very slight, though measurable. The error amounts sometimes to about a half per cent. of the wave length. The arrangement can be understood from Fig. 2. The galvanometer M is placed in a camera as shown. This is provided with a slit in the end, through which parallel light is sent. The ribbon of light falls on the three mirrors of the galvanometer, and is reflected to the cylindrical lens *CL*. This renders the parallel ribbon of light a point. The lens is focussed so that this point is in the plane of the photographic plate. The chute through which the plane is dropped is about ninety-five centimeters long, giving the plate a speed which allows the record of one twenty-five cycle wave on a four by five plate. Light enters the chute through a slit which is provided with the hand shutter *S*. The shutter is open when the plate is dropping, but is closed before the plate is pulled back to the top so that the slide may be inserted. This prevents any possibility of fogging. The plate is carried in an ordinary wooden four by five plate holder. This in turn is held by a light wooden carriage *H*, which is provided with springs on the sides and back. These are adjusted so that they just bear on the surfaces of the chute, thus providing a very steady motion of the plate during the fall. The springs also serve the purpose of holding the front of the plate holder tight against the chute. The plate is started by means of a bulb release *R*, and is stopped by an airdash pot *D.P.* The bottom of the carriage is provided with a leather packed brass piston which fits the cup. Light is provided by a powerful 25-ampere arc light, which has the usual condensers. In addition, to get a proper parallel beam, the concave lens *L* is provided. The whole camera is provided with leveling screws, in addition to those for the galvanometer. This is necessary, since the chute must be vertical. The low potential currents used in this work were brought to the galvanometer by lamp cords, which passed through corks in the side of the box. For convenience in lifting the plate holder and carriage the sliding bracket *F* is provided. This is raised by the knob and string *T*, lifting the carriage until it catches in the release apparatus. The bracket then drops to the bottom of the box. It is held in place by a rod, on which it slides. For access to the camera, the whole top of the box is arranged to slide in a light tight groove.

To adjust the apparatus for a curve the chute is first made perpendicular by the outside levelling screws. Then the galvanometer is levelled with its own screws. The arc light is then adjusted so that a strong beam of closely approximate parallel light falls on the mirrors. The galvanometer is then adjusted, if need be, so that the images pass through the center of the slit. Since the mirrors have practically the same horizontal axis, though they are not in the same vertical plane, necessarily, the spots when focussed on the plate will be in the same horizontal line. They must be adjusted, however, until they have the same vertical axis when not vibrating. This adjustment, as well as the focussing, is done by means of a small glass cylinder with a ground glass end. This is entered through the back of the chute, and is of such a length that the ground glass is in the plane of the falling plate. The focussing is finished and the cylinder removed. After a time, that is with some experience, the focussing can be done from the front of the box. There is no adjustment for the verticality of the mirrors. The coils may be twisted, however, so that they may be brought to various horizontal positions by adjusting screws on the side of the standard.

The double oscillograph permits the simultaneous taking of two independent curves. Since the loops are so fine, and the area of the field so large, comparatively, the loops move in a constant field. The amplitude of the wave is therefore proportional to the maximum value of the current passing through the loops. This will be so only when the damping is critical, that is, sufficient to prevent running past the static position for the same current, and not too much to prevent the loop reaching its static position. A proper adjustment of a non-inductive resistance in series with the loops would make the deflection a definite fraction of an ampere per millimeter. One of the suggested uses to which the double instrument may be put is to obtain the current and potential difference curves for the same piece of apparatus. That this cannot be done, at least exactly, will be apparent from a little study of the conditions. The problem is similar to the wattmeter problem, in which there is always a slight error, due to either the current or the E.M.F. required for one of the coils. The inference is also frequently suggested that the difference in phase, as indicated by the record of the two curves, is the true difference of phase between the quantities. That this is never true, and seldom approximately so, will be apparent from the following discussion :

The simplest method of putting the oscillograph in circuit is shown in Fig. 3. L is the apparatus for which the current and E.M.F. curves are to be determined. O_1 and O_2 are the two loops of the oscillograph joined in circuit as shown. Usually it would be necessary to have a non-inductive resistance in series with each loop to cut down the current to a proper value. A non-inductive shunt is also frequently used with the current loop. Fig. 4 pro-

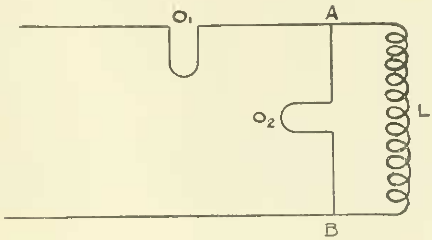

FIG. 3.

vides an analysis of the quantities involved. Let E be the potential difference between A and B. Lagging behind this at an angle α is the current in $L(i)_3$. Also lagging behind E at an angle δ is the current in $O_2(i_2)$. But the current through O_1 is the vector sum of i_2 and i_3 or i_1. Now the angle, or rather the space on the

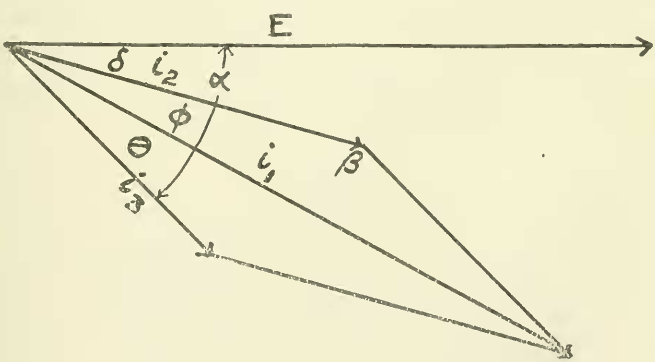

FIG. 4.

plate equivalent to the angle, is of course the angle of phase between i_1 and i_2 and not between E and i_3, as is frequently inferred. It may approach in certain cases, but it is never the true value. The angle desired, that is α, is given by the relation

$$\alpha = \delta + \varphi + \theta. \qquad (1)$$

To obtain it we have the following derivation:

$$\frac{i_1}{i_2} = r = \frac{\sin \beta}{\sin \theta} = \frac{\sin (\varphi + \theta)}{\sin \theta},$$

$$r = \sin \varphi \cot \theta - \cos \varphi,$$

$$\cot \theta = \frac{r}{\sin \varphi} - \cot \varphi,$$

$$\therefore \ \alpha = \cot^{-1}\left(\frac{r}{\sin \varphi} - \cot \varphi\right) + \varphi + \delta. \qquad (2)$$

Before we can determine α, therefore, we have to determine δ.

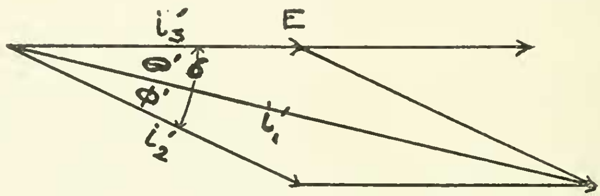

FIG. 5.

The latter may be obtained by putting a non-inductive resistance in place of L. This is shown in Fig. 5, from which as before

$$\delta = \cot^{-1}\left(\frac{r'}{\sin \varphi'} - \cot \varphi'\right) + \varphi', \qquad (3)$$

where

$$r' = \frac{i_1'}{i_2'}.$$

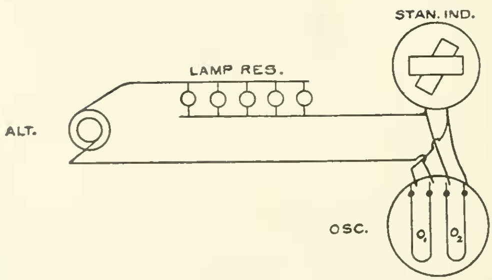

FIG. 6.

To illustrate, and provide a test, arrangements were made as in Fig. 6. An alternator which was under the control of the operator, so that its speed could be maintained constant, was connected through a lamp resistance, a variable standard of inductance, and the loop O_1, in series. The other loop was connected across the inductance. The resistance of the standard of inductance was 9.89 ohms in all positions. The angle of lag for any given position was therefore easily calculated. Moreover, as the resistance of an oscillograph loop with its fuse, is also about ten ohms, quite a difference between a and φ could be expected. Now we have to determine r, φ, and δ, in order to determine a. The procedure was as follows. With the inductance set to some definite value, L, a plate was taken. Such a plate is shown by Fig. 7. With

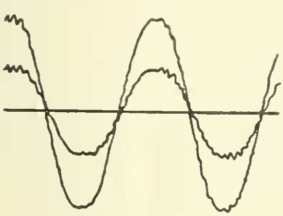

FIG. 7.

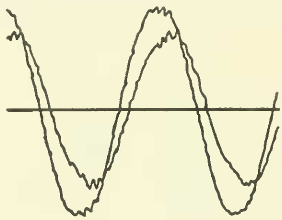

FIG. 8.

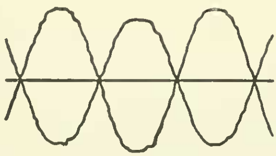

FIG. 9.

the standard set to zero another plate was taken. This is shown in Fig. 8. Now to obtain the ratio r, we have to measure the ratio of the amplitudes of the two waves. As these waves are obtained from two different loops, it will be necessary to obtain the ratio of the galvanometer constants of the two loops. A third plate was taken with the two loops connected in series, and one current passing through both. For accuracy in measurement, the loops were connected oppositely, so that an apparent phase difference of 180 degrees results. This is shown in Fig. 9. The following measurements are then made. $C =$ the ratio of galvanometer constants of the two obtained from ratio of amplitudes in Fig. 9.

$r = i_1/i_2$ obtained from ratio of amplitudes in Fig. 7.

λ_ϕ and $\lambda_{\phi'}$ wave-lengths obtained from Figs. 7 and 8 respectively.

l_ϕ and $l_{\phi'}$ phase displacement, obtained from Figs. 7 and 8 respectively r obtained from ratio of amplitudes in Fig. 7.

During the taking of each plate the value of the frequency must be observed.

R = resistance of the standard of inductance. We then have the following:

$$\varphi = \frac{l_\phi}{\lambda_\phi} 360, \quad \varphi' = \frac{l_{\phi'}}{\lambda_{\phi'}} 360.$$

By substituting φ' and r' in (3) we get δ. By substituting φ, δ and r in (2) we get α. The true value of α is given by the relation

$$\alpha = \tan^{-1} \frac{L\omega}{R}.$$

Values for α as obtained from five different sets of plates are given. A calculation on the first plate showed that δ was practically equal to φ', so that (3) was not used.

No.	C	λ_ϕ	l_ϕ	r	$\lambda_{\phi'}$	$l_{\phi'}$	δ	ϕ degrees.	θ degrees.	α degrees.	L henrys.	$\tan^{-1} \frac{L\omega}{R}$ degrees.
1	1.14	1.74	.11	1.36	1.76	.025	5.1°	22.8	32.3	59	.040	56.8
2	1.15	1.79	.10	1.55	1.73	.025	5.2	20.0	23.5	47	.030	48.8
3	1.12	1.98	.125	1.36	2.02	.020	3.6	22.3	31.8	59	.040	56.8
4	1.13	2.17	.13	1.36	2.11	.025	4.3	21.6	30.5	57	.040	56.8
5	1.12	2.08	.13	1.37	2.12	.025	4.2	22.3	31.8	59	.040	56.8

$R = 9.89 \qquad \omega = 377$

It is evident, that in the simplest application of the oscillograph, a mere inspection of the plate is not sufficient to obtain the true difference of phase. The general case is much more complex. This is represented by the diagram of connections in Fig. 10, and the vector diagram in Fig. 11. It is necessary to introduce a series non-inductive resistence in circuit with O_2, and a shunt on O_1, in order to bring the current to a proper size. The letters have the same significance as before, and in addition s is the resistance of the shunt, R' is the series resistance, i_t is the total current, and i_s is the current in the shunt. As before, the angle (Ei_3) is the quantity desired. Now a plate obtained with the connections arranged as in Fig. 10, would give us i_1, i_2, and the angle (i_1i_2), granting that the loops have been calibrated so that current values may be measured by them. It is evident that if the multiplying power of the shunt for direct currents is known, i_t can be obtained from i_1 without serious error. Also since s is non-inductive, the angle $(i_1 i_3)$ can be known from a plate taken similarly to Fig. 8. Then the angle $(i_1 i_t)$ can be calculated, and subtracted from the angle $(i_1 i_2)$ leaving the angle $(i_2 i_t)$. Having the latter angle, with i_t and i_2, $(i_t i_3)$ can be calculated, and added to $(i_t i_2)$, and (Ei), which is known as before, thus giving Ei_3, which is the desired angle.

The writer has not worked the latter discussion out for two reasons. First, the errors of measurement on a photographic plate

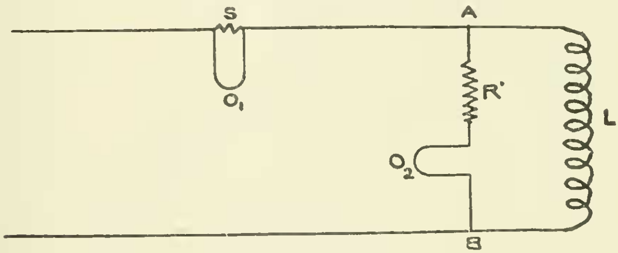

Fig. 10.

would not permit of any accurate results since in any case the angles are small. Second, in the large majority of cases, the multiplying power of the shunt is so large, that the currents i_1 and i are in the same phase, so far as could be measured. Also the resistance in the O_2 circuit so large that E and i are in the same phase. Therefore the angle desired can be obtained exactly as δ was in the preceding discussion, with the introduction of the multiplying power of the shunt. The use of ammeters in some of the circuits would greatly facilitate matters in many cases.

The above discussion shows that, with the oscillograph, the phase angle can be calculated. In no case can it be obtained by a simple measurement on a single plate. Owing to the thickness of the lines traced by the moving spot, measurements cannot be made closer than a quarter of a millimeter, so that a long curve must be obtained if any accuracy is to be attained, especially with small differences of phase. While the double oscillograph could not be recommended in any way as a phase meter, yet it does permit an ap-

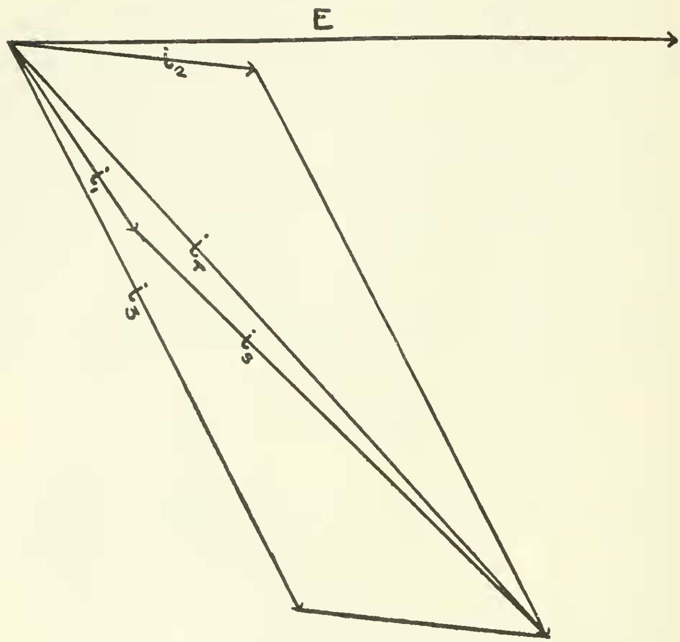

FIG. 11.

proximate value of this quantity being obtained. In most of the cases to which the instrument has been applied, the process would be quite simple, owing to the magnitude of the quantities involved. The fact that the angle can be obtained is valuable in those cases where curves are taken under fleeting or unknown conditions, in which other methods could not be applied.